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A method for chronological apportioning of ceramic assemblages

John M. Roberts Jr. [a,*], Barbara J. Mills [b], Jeffery J. Clark [c], W. Randall Haas Jr. [b], Deborah L. Huntley [c], Meaghan A. Trowbridge [b]

[a] Department of Sociology, MSC05 3080, University of New Mexico, Albuquerque, NM 87131, USA
[b] School of Anthropology, University of Arizona, PO Box 210030, Tucson, AZ 85721, USA
[c] Archaeology Southwest, 300 North Ash Alley, Tucson, AZ 85701, USA

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Artifact assemblages from long-inhabited sites may include ceramic types and wares from multiple time periods, making temporal comparisons between sites difficult. This is especially problematic in macro-regional data sets compiled from multiple sources with varying degrees of chronological control. We present a method for chronological apportioning of ceramic assemblages that considers site occupation dates, ceramic production dates, and popularity distribution curves. The chronological apportioning can also be adjusted to take into account different population sizes during the site occupation span. Our method is illustrated with ceramic data from late prehispanic sites in the San Pedro Valley and Tonto Basin, Arizona, U.S.A., compiled as part of the Southwest Social Networks Project. The accuracy of the apportioning method is evaluated by comparing apportioned assemblages with those from nearby contemporaneous single component sites.

1. Introduction

When artifact assemblages—frequency distributions of recovered objects grouped into classes (e.g., ceramic types and wares)—are not chronologically differentiated, assemblages associated with a long-inhabited site may be attributable to multiple components. Such mixture problems (Kohler and Blinman, 1987) are ubiquitous in archaeology, as often there is insufficient stratigraphic control to associate particular objects with particular periods. The problem becomes particularly acute in macro-regional data sets compiled from multiple sources with differing degrees of chronological precision, and will become increasingly important as archaeologists compile data from multiple projects to address research questions at larger spatial scales.

We confronted this problem in our own work on the Southwest Social Networks Project, which has compiled ceramic, obsidian, and architectural data from hundreds of sources across the Western U.S. dating between A.D. 1200 and 1550. Our ultimate goal is to use these data to examine changing interaction networks. Without some means of apportioning artifacts by time, however, assemblage mixing precludes comparisons of different sites' contemporaneous occupations. Previous research on the quantitative analysis of assemblages and time has focused on estimating site occupation spans (Carlson, 1983; Ortman, 2003; Steponaitis and Kintigh, 1993). We draw on this work, but are interested in a somewhat different situation, in which the site occupation spans are largely known and the goal is to apportion recovered assemblages into discrete chronological periods.

The method we present for chronological apportioning uses known information on site occupation span, periods of artifact production, and hypothesized artifact popularity curves. If an artifact assemblage was obtained from, as was typical, a multi-component site with changing population size, the method can also incorporate the settlement's demographic history. We illustrate the method with ceramic assemblage data from late prehispanic (A.D. 1200–1400) sites in Arizona's San Pedro River Valley and Tonto Basin. To evaluate the method's effectiveness, we select pairs of geographically close sites that "should" be culturally similar, and therefore should have similar ware distributions during intervals of temporal overlap. Further, in each pair one site is a single component or at least single period occupation and the other is a multi-component site whose occupation includes the date range of the former. This allows comparison of apportioned mixed assemblages from multi-component sites with unmixed assemblages as test cases.

2. Related methods

The approach to estimating site occupational duration developed by Carlson (1983) and Steponaitis and Kintigh (1993) relies on

* Corresponding author. Tel.: +1 505 277 3940; fax: +1 505 277 8805.
E-mail address: jmrob@unm.edu (J.M. Roberts).

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a known lifespan of each recovered ceramic type's production or use and a hypothesized distribution of its popularity over that lifespan. While the goal of estimating site occupation spans using a composite sherd distribution differs from ours (that is, apportioning total sherds into specific periods from a site with a known occupation span), both rely on distribution curves that approximate a ceramic type or ware's popularity and use life. Use of an empirical "calibration data set" from single component sites is an alternative to a hypothesized popularity distribution. Kohler and Blinman (1987) used such a calibration data set to estimate the proportions of a multi-component site's total ceramic ware assemblage that came from different periods. Kohler and Blinman's calibration data combined local site assemblages that could be dated accurately by tree-ring, stratigraphic, or architectural evidence. Ortman's (2003) construction of a composite distribution (for estimating inhabitation periods) also relied on a calibration data set, with each ware or type's period probabilities estimated by adjusting data on the composition of each period's combined assemblage in the calibration data set. Ortman et al. (2007) further refined that approach.

These works highlight the choice between describing ceramic types or wares' historical popularity through theoretical distributions or empirically derived calibration data sets. We use theoretical distributions below, as our research project encompasses many areas lacking concentrations of short-lived sites needed to construct calibration data sets, particularly for the time periods of interest in our research. Also, Ortman et al. (2007) adjusted empirically derived distributions to be unimodal, and many of their resulting distributions seem similar to those hypothesized by Carlson (1983) and Steponaitis and Kintigh (1993). Note, though, that our framework is compatible with empirically derived distributions, so that this choice can reflect context, scale, and research questions. While we focus on the uniform and truncated normal distributions below, we do not claim that these will be the best choices in all settings.

3. Apportioning method

Our apportioning method requires start/end dates for wares' lifespans, which are derived from refined chronometric information on U.S. Southwest ceramic types that make up each ware. For each ceramic type and ware, we compiled production spans used in the archaeological literature from throughout the Southwest (e.g. Goetze and Mills, 1993). Although the database retains typological identifications, we use the larger technological category of wares because (1) types are more prone to identification errors, and (2) our main interest is in comparisons at the regional scale where ware differences are more relevant. We also require start/end dates for the sites' inhabitation; these dates are likewise available in this setting, based on data compiled by numerous researchers addressing Southwest Late Prehispanic sites (e.g., Hill et al., 2004). For apportioning, we divide a site's inhabitation span into a series of periods. We focus on 25- or 50-year periods; in principle, the method can accommodate any period length, but apportioning into shorter periods may lead to false precision.

For each ware we require a known or hypothesized function or curve representing the ware's relative popularity, or level of production and use, across its lifespan, conceptualized as a probability density function over the ware's lifespan with cumulative distribution function $F(x) = P\{X \le x\}$. We follow Carlson (1983) and Steponaitis and Kintigh (1993) in using a theoretical continuous distribution, but F could also be empirically derived and/or discrete. The entire curve represents the ware's entire production or use history (associated with probability 1), and any period's proportion is indicated by the area under the curve for that period. If a ware first appeared in A.D. 1140 and was last produced in 1320, then $F(1320) = 1$ and $F(1140) = 0$. The share of production that occurred between, say, 1200 and 1250 is given by $F(1250) - F(1200)$. It is simplest to assume the same type of curve for each ware, but the method could in principle accommodate different curves for different wares.

Our method uses the relative popularity of the ware at each period within a site's occupation. Continuing the example above, suppose that the site was occupied from 1200 to 1300, and that our interest is in apportioning this ware's sherds into the periods 1200–1250 and 1250–1300. We focus our attention on the period of overlap between the ware's history and the site's inhabitation, 1200–1300 in this example, and the area under the curve for that overlap period. The share of sherds apportioned to the period 1200–1250 will be the area under the curve between 1200 and 1250 divided by the area under the curve for the entire overlap period (1200–1300). Likewise, the share apportioned to the period 1250–1300 will be the area under the curve between 1250 and 1300 divided by the area under the curve for the entire overlap period (1200–1300). In other words, the share of sherds apportioned to 1200–1250 is $[F(1250) - F(1200)]/[F(1300) - F(1200)]$, and the share apportioned to 1250–1300 is $[F(1300) - F(1250)]/[F(1300) - F(1200)]$. The denominators reflect the restriction of attention to the portion of the ware's history that overlaps with the site's inhabitation.

In this way the apportioning of the total sherd count into periods of the site's inhabitation follows the ware's relative popularity in those periods of its history. In practice, an additional adjustment to this apportioning may be appropriate; we defer that discussion for now. In a slightly different perspective, these proportions of the total count can also be thought of as the probabilities of a single sherd being assigned to 1200–1250 or 1250–1300. In this perspective, the apportioned counts indicate the expected numbers of individual sherds assigned to the various periods. If N_j indicates the site's number of sherds of ware j, and p_{jt} is the probability that a sherd of ware j is apportioned to period t, expected apportionings to the various periods are given by the products $N_j p_{jt}$. Typically the values $N_j p_{jt}$ will be non-integers; for an integer summary apportioning, one could simply round off the $N_j p_{jt}$ (being careful to maintain the sum N_j), or report the integer apportioning that has the maximum probability under the estimated p_{jt}.

Of course a site's inhabitation may not be completely contained within the ware's history, perhaps beginning before the ware existed or ending after the ware's lifespan. In this example, suppose that the ware's end date is 1250, not 1320. Then all of the site's sherds of this ware would be apportioned to the 1200–1250 period, and none to 1250–1300. Whatever the nature of this overlap, the apportioning is based on this transformation of the ware's popularity distribution, which in effect produces the conditional distribution defined by restricting the ware's popularity distribution to the site's occupational history. The Appendix formally describes our approach, and Fig. 1 illustrates the method graphically for a site occupied between 1150 and 1300 and two wares, one produced between 1150 and 1350 and the other produced between 1200 and 1400, whose historical popularity is represented by a normal curve.

3.1. Choice of distribution

One possible ware popularity curve is the uniform distribution, graphed as a flat line over the ware's history. Carlson (1983) considered the uniform distribution; it assumes constant popularity, so that the share of production or use attributable to any given period is simply that period's proportion of the ware's history. Conceptually this is similar to the unapportioned assemblage, but it

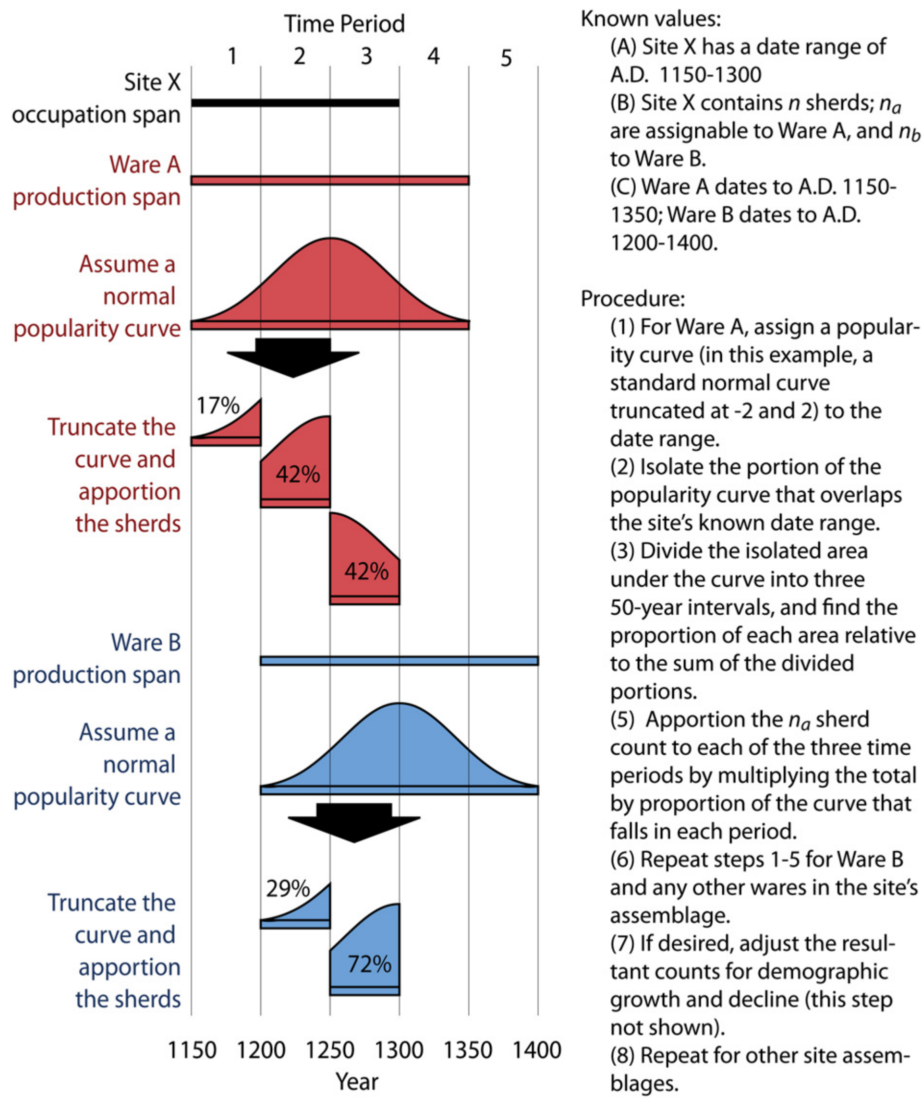

Fig. 1. Illustration of apportioning method.

will differ if some wares' lifespans do not cover the site's entire inhabitation.

The flat/uniform distribution's assumption of constant popularity over a ware's history may be less realistic than a trajectory of rising and falling popularity. That could be represented by the familiar curve of the standard (mean 0, variance 1) normal distribution. Carlson (1983) also used the normal distribution curve. Note that the curve's tails extend to infinity in each direction, while ware history is finite and the popularity distribution is restricted to the years in this finite lifespan. This requires truncation of the distribution, but the choice of truncation point affects apportioning by changing the ware's hypothesized relative popularity early and late in its lifespan (Carlson, 1983). In applications we have followed Carlson by truncating at 2 and −2 standard deviations from the mean 0. Of course there are other possible distributions; Carlson (1983) considered the chi-square distribution, Steponaitis and Kintigh (1993) the gamma, and in our work we have examined the beta family. As noted above, different distributions may be more or less appropriate in different settings. But absent any further theoretical guidance, the uniform and normal are reasonable starting points and, for our purposes here, helpful as examples illustrating our method.

3.2. Small example

Below we discuss an extended example using ceramic assemblage data from Bayless Ranch Ruin, a San Pedro River Valley site that was inhabited A.D. 1200 to 1350 at 50-year resolution (Clark and Lyons, 2012). Before considering the full example, it is useful to carefully examine the apportioning of one ware, Corrugated Brown Ware. Corrugated Brown Ware's lifespan (A.D. 800–1400) included Bayless Ranch Ruin's entire inhabitation, and we wish to apportion its 333 sherds into three 50-year periods (1200–1250, 1250–1300, and 1300–1350). The period of overlap between site occupation and ware history is 1200–1350 (in this case, the site's entire occupation period). Therefore we restrict attention to the area under the ware's popularity history curve between 1200 and 1350.

For the flat/uniform distribution, this period of overlap includes 25% of the area under the entire ware history curve (because the 150-year overlap period represents 25% of the entire 600-year history of Corrugated Brown Ware). Each 50-year period likewise represents 8.33% of the area under the entire ware history curve, which means that the apportioned shares for each of 1200–1250, 1250–1300, and 1300–1350 are 0.0833/0.25 = 0.333. That is, when there is no change in the ware's popularity over the years, as implied by the flat/uniform distribution, each 50-year period of site-ware overlap receives the same apportioning. In this case, each 50-year period is assigned 33.3%, or 111, of Bayless Ranch Ruin's 333 Corrugated Brown Ware sherds.

We can also characterize Corrugated Brown Ware's history with the truncated (at −2 and 2) standard normal distribution. (This apportioning is described more formally in the Appendix.) The truncation values establish that one unit in this distribution is equivalent to 150 years; aside from this, we can ignore the truncation in the subsequent apportioning calculations, as the same adjustment due to truncation will apply to numerator and denominator in all fractions. Ignoring the adjustment for truncation, the normal table indicates that the period of overlap (1200–1350) represents 20.5% of the area under the entire history curve. The periods 1200–1250, 1250–1300, and 1300–1350 account for areas representing 9.38%, 6.75%, and 4.34%, respectively, and in turn the apportioning shares are $0.0938/0.205 = 0.458$, $0.0675/0.205 = 0.329$, and $0.0434/0.205 = 0.212$. The 333 Corrugated Brown Ware sherds are therefore apportioned to the three periods as 152.51, 109.56, and 70.60 sherds, respectively. (Note that when using more precise figures for areas under the curve, and the fractions above, the apportioned sherds will in turn sum more precisely to 333.)

3.3. Adjustment for demographic history

The amount of use (and discard) of ceramic vessels likely reflects, at least roughly, population size, although this relationship is complex and may vary for different ceramics (Mills, 1989; Pauketat, 1989; Sullivan, 2008; Varien and Mills, 1997). Apportioning results can be adjusted to match changing population size to each period's total count of a comprehensive set of wares. With the apportioned counts $N_j p_{jt}$ summarized in a $J \times T$ matrix, column sums (for period t, summing $N_j p_{jt}$ over the J wares) give period totals. Hill et al. (2004) estimate site population at a given period across the prehispanic Southwest using room count data and an inferred site population curve that is skewed to reflect slow population growth and fairly rapid depopulation. We want to adjust the ceramic apportioning so that a period's apportioned sherd total is proportional to its population estimate.

Classical iterative proportional fitting (Deming and Stephan, 1940) adjusts the row and column totals of a cross-classification table to a set of desired totals while preserving (in at least one sense) the original table's interaction between row and column variables. The procedure first multiplies all cell counts in the first row by the factor required to achieve the desired first row total, and repeats for all rows. Next such multipliers are applied to all the columns; these column adjustments will likely have disturbed the row totals, so each row must be readjusted. Row and column adjustments continue until all row and column totals are sufficiently close to those desired. The resulting table may look very different from before, but all odds ratios—key indicators of row-column association—are preserved. The apportioning matrix's odds ratios express the likelihood of a sherd of ware j being assigned to period t rather than period t' relative to this likelihood for a sherd of ware j'. Bringing period totals into line with population estimates this way does not change these fundamental relations between wares.

The iterative proportional fitting adjustment can be used in conjunction with both theoretically- and empirically-derived population estimates, so it would be possible to examine alternatives to the Hill et al. (2004) estimates that we employ below. Also, there may be interest in using the unadjusted period totals from the original apportioning to produce ceramic-based population estimates. Population estimates of that sort would be akin to the site occupation span estimates of Carlson (1983) and Steponaitis and Kintigh (1993), and could be compared to other empirical or theoretical estimates. (We do not pursue such estimates or comparisons here.)

Note that in practice it may be impossible to so adjust the apportioning matrix. When a site has a large sherd count for a ware whose lifespan overlapped with the site's inhabitation for only one period, that ware's entire count is assigned to that period. If this count exceeds the desired total for that period, iterative proportional fitting will fail, because it cannot reduce that ware's contribution to the period total. Our experience thus far is that this is unusual; it did not occur for any San Pedro Valley or Tonto Basin sites, but we have encountered it in other data. In such cases we have simply retained the original apportioning and noted that adjustment to the population estimates was not possible.

4. Example

Table 1 provides the Bayless Ranch Ruin assemblage data, including sherd counts and each ware's lifespan (Clark and Lyons, 2012). Table 1 excludes 47 sherds of wares found at the site but whose lifespans were outside the primary site occupation. We apportion the Bayless Ranch Ruin assemblage into three 50-year periods (1200–1250, 1250–1300, and 1300–1350).

Table 2 gives the apportioning of the entire assemblage, under both the flat/uniform and truncated (−2, 2) standard normal distributions. (Figures for Corrugated Brown Ware under the truncated standard normal differ slightly from those above, due to less rounding when calculating areas under the curve.) While each row total is the total number of the corresponding ware's sherds, note the column totals, or the total apportioned counts at each period. These simply result from the separate apportioning of each ware, rather than reflecting any theoretical or other empirical estimate of population size. As the complete assemblage is apportioned, it is reasonable for the total apportioned counts to reflect the site's demographic history. Hill et al.'s (2004) method estimates relative population sizes of 0.7, 1.0, and 0.7, respectively, in the three periods, so that estimated population was greatest in 1250–1300, and smaller (and equal) in 1200–1250 and 1300–1350. Neither distribution's apportioned period totals in Table 2 reflect this: the flat/uniform approach has the largest period total in 1300–1350, while the normal approach shows ceramic deposits as greatest in 1200–1250.

The estimated relative population sizes suggest sherd totals of 666.75, 952.5, and 666.75 for the three periods (2286 sherds distributed according to relative population sizes 0.7, 1.0, and 0.7). Table 3 adjusts both of Table 2's apportionings to these totals via iterative proportional fitting. Table 3's row totals are unchanged from Table 2, but the column totals now reflect the site's demographic history. Again, this adjustment is not guaranteed to succeed, but it has in almost all of our applications thus far.

Table 1
Ceramic assemblage at Bayless Ranch Ruin.

Ware	Sherd count	Ware start date	Ware end date
Cibola White Ware	4	550	1325
Corrugated Brown Ware	333	800	1400
Late Middle Gila Red Ware (plain & red-slipped)	11	1200	1450
Late Tucson Basin Red-on-brown Ware	11	1150	1300
Maverick Mountain Series	13	1275	1400
Plain Brown Ware	1605	200	1450
Roosevelt Red Ware	252	1275	1450
San Carlos Brown Ware	9	1200	1450
Tucson Basin Brown Ware	48	750	1300

Table 2
Apportioning of ceramic assemblage at Bayless Ranch Ruin under flat/uniform and truncated (−2, 2) standard normal distributions, prior to demographic adjustment.

Ware	Period		
	1200–1250	1250–1300	1300–1350
Flat/Uniform			
Cibola White Ware	1.600	1.600	0.800
Corrugated Brown Ware	111.000	111.000	111.000
Late Middle Gila Red Ware (plain & red-slipped)	3.667	3.667	3.667
Late Tucson Basin Red-on-brown Ware	5.500	5.500	–
Maverick Mountain Series	–	4.333	8.667
Plain Brown Ware	535.000	535.000	535.000
Roosevelt Red Ware	–	84.000	168.000
San Carlos Brown Ware	3.000	3.000	3.000
Tucson Basin Brown Ware	24.000	24.000	–
Total	683.767	772.100	830.133
Truncated Standard Normal			
Cibola White Ware	2.115	1.397	0.488
Corrugated Brown Ware	152.690	109.718	70.592
Late Middle Gila Red Ware (plain & red-slipped)	1.604	3.992	5.404
Late Tucson Basin Red-on-brown Ware	7.513	3.487	–
Maverick Mountain Series	–	1.896	11.104
Plain Brown Ware	658.990	530.189	415.821
Roosevelt Red Ware	–	37.146	214.854
San Carlos Brown Ware	1.313	3.266	4.421
Tucson Basin Brown Ware	30.870	17.130	–
Total	855.095	708.222	722.683

5. Evaluation

We selected several test cases to evaluate the apportioning method's effectiveness. The logic of the evaluation is as follows. From our work with San Pedro Valley and Tonto Basin data, we identified pairs of sites in which the two sites, A and B, were geographically close and likely culturally similar, *and* in which A's habitation history was shorter than (ideally only one 50-year interval) and contained within B's. We focused on the period of overlap. A's assemblage would all be attributed to that period, while for the longer-lived B there would be an apportioning to both this focal period and others. The comparison between the total (for A) and apportioned (for B) assemblages in the focal period provides an interesting test, because the sites' geographical proximity and cultural similarity would suggest similar assemblages during the interval of overlap. While there is no explicit theoretical guidance as to exactly how similar the two assemblages should be—this likely depends on a host of social and economic factors that vary across different settings—a relative approach to assessing the apportioning method would be to see whether the similarity between A's total assemblage and B's chronologically apportioned assemblage is greater than the similarity between A's assemblage and B's unapportioned total assemblage. If B's apportioned assemblage is more similar to A's than is B's unapportioned assemblage, the apportioning is likely more accurate than simply applying the total assemblage to B's entire inhabitation.

Table 4 shows the pairs of sites. The San Pedro assemblages were generally from temporally mixed middens and the Tonto Basin assemblages were from a combination of structures, middens and pits. In each pair, we compared the first site's total assemblage with the second site's apportioned assemblage for the period of the first site's inhabitation.

Table 3
Apportioning of ceramic assemblage at Bayless Ranch Ruin under flat/uniform and truncated (−2, 2) standard normal distributions, with demographic adjustment.

Ware	Period		
	1200–1250	1250–1300	1300–1350
Flat/Uniform			
Cibola White Ware	1.504	1.888	0.608
Corrugated Brown Ware	108.689	136.399	87.912
Late Middle Gila Red Ware (plain & red-slipped)	3.590	4.506	2.904
Late Tucson Basin Red-on-brown Ware	4.878	6.122	–
Maverick Mountain Series	–	5.679	7.321
Plain Brown Ware	523.864	657.417	423.719
Roosevelt Red Ware	–	110.090	141.910
San Carlos Brown Ware	2.938	3.686	2.376
Tucson Basin Brown Ware	21.287	26.713	–
Total	666.750	952.500	666.750
Truncated Standard Normal			
Cibola White Ware	1.659	1.890	0.451
Corrugated Brown Ware	119.625	148.226	65.150
Late Middle Gila Red Ware (plain & red-slipped)	1.188	5.098	4.714
Late Tucson Basin Red-on-brown Ware	6.110	4.890	–
Maverick Mountain Series	–	2.600	10.400
Plain Brown Ware	512.667	711.258	381.075
Roosevelt Red Ware	–	50.896	201.104
San Carlos Brown Ware	0.972	4.171	3.857
Tucson Basin Brown Ware	24.529	23.471	–
Total	666.750	952.500	666.750

Table 4
Pairs of sites for evaluating the apportioning method.

San Pedro Valley
Wright (1300–1400) vs. Ash Terrace (1200–1400)
Big Pot (1200–1250) vs. Lost Mound (1200–1350)
Buzan (1200–1300) vs. Lost Mound
Tonto Basin
Cline Terrace (1300–1400) vs. Casa Bandolero Complex (1200–1400)
Cline Terrace vs. U:3:128 (1200–1400);
Schoolhouse Point Mesa Complex (1200–1300) vs. Schoolhouse Point Mound (1200–1400);
U:8:515 (1250–1300) vs. Armer Gulch Ruin (1250–1400);
U:8:515 vs. U:8:589 (1200–1300).

Inhabitation dates in parentheses.

We used three different ware popularity curves: (i) flat/uniform; (ii) truncated standard normal at 2 and −2; and (iii) truncated standard normal at 3 and −3. For reference, we also compared the total unapportioned assemblage at the second site to the total assemblage at the first. Each apportioning included demographic adjustment. We compared separately for "decorated" and "plain/red/corrugated" (PRC) wares, because the greater quantity of PRC in the entire assemblages could swamp results for the fewer decorated wares, and the two classes of wares likely were produced, exchanged, and used in different socioeconomic contexts. We did not compare decorated wares for Big Pot versus Lost Mound or Cline Terrace versus Casa Bandolero due to the low frequencies recovered from these sites. We also removed many essentially meaningless undifferentiated wares, and did not use wares whose date ranges did not overlap a site's occupation. For San Pedro sites, we distinguished 19 decorated and 6 PRC wares, and 20 and 8, respectively, for Tonto Basin sites.

The assemblage comparisons used the dissimilarity index *D*. If N_{ij} represents the number of ware *j* sherds at site *i*, and N_{i+} represents the total number of sherds (of all wares) at site *i*, the dissimilarity index comparing sites *i* and *k*'s assemblages is

$$D_{ik} = \sum_j \left| \left(N_{ij}/N_{i+}\right) - \left(N_{kj}/N_{k+}\right) \right| \Big/ 2.$$

This measure ranges from 0 to 1, with larger values indicating more dissimilarity (less similarity). It is formally equivalent to the familiar Brainerd-Robinson statistic (Brainerd, 1951; Robinson, 1951; Cowgill, 1990); although Brainerd-Robinson ranges from 0 to 200, with larger values indicating more similarity, the measures are functions of each other. *D* values show what proportion of one site's sherds would need to be shifted to different ware categories to match the other site's (proportional) assemblage.

Table 5 reports the assemblage comparisons. The first row, for example, compares Wright and Ash Terrace using decorated wares. The dissimilarity index between the total assemblages, including the unapportioned assemblage for Ash Terrace, is 0.101. When the Ash Terrace assemblage is chronologically apportioned using the flat/uniform ware distribution, the dissimilarity between the total Wright (decorated) assemblage and Ash Terrace's apportioned assemblage for 1300–1400 is 0.041. Using the truncated standard normal curve with truncation at 2 yields $D = 0.052$, and truncating at 3 gives $D = 0.063$. The apparent similarity between Wright and Ash Terrace in 1300–1400 under any of these apportioning methods is greater than the two total assemblages' similarity. As these sites are thought to be culturally similar, Ash Terrace's apportioned assemblage may better represent the empirical reality in 1300–1400 than simply assuming that the unapportioned assemblage applied throughout all periods. Note that in this case the simpler flat apportioning yielded a smaller dissimilarity index than did either normal-based approach.

The various comparisons are not statistically independent, particularly as some sites appear more than once, and so limit our interpretation to some informal observations. As decorated wares may be of greater interest than PRC in many research settings, we consider the two sets of comparisons separately. In 5 of the 6 decorated comparisons, at least one apportioning yields a smaller dissimilarity index than was obtained in the comparison of the unapportioned assemblages. Three of the 6 decorated comparisons showed a smaller *D* for the truncated (at 2) standard normal approach than for the flat/uniform, and in only 1 of the 6 cases did truncation at 3 yield a smaller *D* than did truncation at 2. In the 8 comparisons involving PRC wares, the flat/uniform apportioning gave the same *D* values (in one case not exactly, but still at least to three decimals) as the raw comparison of total assemblages. This is due to the PRC wares' long use lives spanning entire site occupations. Among the apportioning methods, the truncated standard normal yielded a smaller *D* than the flat/uniform (and in turn the raw total assemblage) in only 3 of 8 instances. *D* was smaller for truncation at 2 than at 3 in 6 of the 8 comparisons.

Table 5
Dissimilarity Indices *D* from comparisons of apportioned assemblages with observed test site assemblages.

Sites	Observed or apportioned comparison site assemblage			
	Observed	Flat/Unif.	Normal (−2, 2)	Normal (−3, 3)
Decorated Wares				
Wright vs. Ash Terrace	0.101	0.041	0.052	0.063
Buzan vs. Lost Mound	0.193	0.168	0.257	0.420
Cline Terrace vs. U:3:128	0.040	0.126	0.149	0.165
Schoolhouse Point Complex vs. Schoolhouse Point Mound	0.581	0.334	0.207	0.215
U:8:515 vs. Armer Gulch Ruin	0.240	0.129	0.039	0.057
U:8:515 vs. U:8:589	0.394	0.369	0.266	0.184
Plain/red/corrugated Wares				
Wright vs. Ash Terrace	0.169	0.169	0.249	0.307
Big Pot vs. Lost Mound	0.323	0.323	0.122	0.031
Buzan vs. Lost Mound	0.253	0.253	0.159	0.137
Cline Terrace vs. Casa Bandolero	0.064	0.064	0.079	0.092
Cline Terrace vs. U:3:128	0.053	0.053	0.076	0.098
Schoolhouse Point Complex vs. Schoolhouse Point Mound	0.155	0.155	0.189	0.224
U:8:515 vs. Armer Gulch Ruin	0.214	0.214	0.263	0.317
U:8:515 vs. U:8:589	0.439	0.439	0.414	0.392

6. Discussion

The availability of chronologically apportioned assemblages for long-inhabited sites makes possible a variety of investigations, and we draw on previous work with different research goals to attack this problem. While the method requires rather rich temporal information on the objects (ceramic wares), in many research settings this is realistic. The adaptability of this approach to any hypothesized or empirical ware popularity curve is also attractive. As theory or past research may give little reason to prefer one type of popularity curve over another, researchers may want to explore results under a number of different possibilities.

Of course a method must produce valid results with real data. To this end, further consideration of the evaluation results is worthwhile. We proceeded from the idea that, in these pairs, more similarity has greater credibility than less, even if it is unclear what precise level of similarity/dissimilarity should characterize the pairs. Under this perspective, apportioning seemed quite successful in the comparisons involving only decorated wares, but this was less clear in the comparisons involving PRC wares. As noted, the PRC wares differ in typically having long lifespans, resulting in equivalences between the (proportional) apportioned assemblages under the flat/uniform approach and the total unapportioned assemblages. So in the PRC comparisons for these data, the choice is really between no apportioning and a non-uniform popularity curve.

While the less than overwhelming performance of the truncated standard normal in the PRC comparisons could reflect idiosyncracies of the particular test cases examined, it may also reflect a genuine difference in the nature of the popularity curves that properly characterize these wares. The utilitarian nature of PRC wares may mean that there was in fact relatively little change in their historical popularity, and so the flat/uniform curve could be a better description of such wares' histories than the truncated standard normal curve. On the other hand, the possibly greater cultural significance of the decorated wares may correspond to a more dramatic rising and falling popularity over the years in which they were produced. If so, a curve like the truncated standard normal could better characterize such wares in general, and produce better apportioning. More investigation would be needed to establish the general superiority of any particular distribution, and, as noted above, conclusions regarding the "best" distribution likely differ across research settings. In any case, we do not regard the evaluation results as unduly pessimistic assessments of the apportioning method's value, and we are now applying the method in our subsequent research. In addition, once apportioned, the frequency data could be converted to more robust interval data (e.g., 1–10% or even quartiles) for use in large-scale regional comparisons of ceramic ware distributions.

Of the two truncated standard normal curves used in the evaluation, results suggest that truncation at 2 (in each tail) may be better than truncation at 3. When more of the tail is included (truncation at 3), the ware's popularity is more different between the early/late parts of its lifespan and the middle. This may be appropriate sometimes, but in more cases here it seemed that a less dramatic variation in popularity was preferable. In our substantive work we have focused on truncation at 2, but again this calls for more investigation, including both theoretical development and empirical evidence (Ortman et al., 2007).

It is important to consider sampling variability in analyses that use the apportioned counts. Space prohibits a full discussion here, but we have used the bootstrap method (e.g. Ringrose, 1992) to assess sampling variability. Bootstrapped data sets are constructed by resampling (with replacement) from the total assemblage at each site. The bootstrapped data set can then be apportioned and analyzed; repeating this many times allows assessment of the sampling distribution of some statistic of interest. Also, here we did not explore the use of different popularity curves for different wares. Our current software is not written for this level of sophistication, but the evaluation results make this a worthwhile topic for future research. In any case, the apportioning method described here may be useful in a variety of substantive domains, and we look forward to learning from future applications.

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Appendix

A. 1. Formal description

We describe apportioning a single site's observed counts of wares $j = 1, \ldots, J$ into the time periods $t = 1, \ldots, T$ of its inhabitation (before any demographic adjustment). Some notation:

s_0, s_1: beginning, ending dates of the site's inhabitation.
d: length of apportioning time periods; for convenience, assume that s_0 and s_1 are multiples of d (so the site's inhabitation length $s_1 - s_0$ is also).
w_{j0}, w_{j1}: beginning, ending dates for ware j; if $w_{j1} \le s_0$ or $w_{j0} \ge s_1$, the ware lies outside the known site occupation or study interval and will not be apportioned. Note that we treat a site ending in 1300, for example, as not overlapping with a ware first appearing in 1300.
$F(x)$: cumulative distribution function describing the ware's historical popularity; $F(w_{j1}) = 1$, $F(w_{j0}) = 0$.
p_{jt}: probability of apportioning a ware j sherd to period t.

Ware j in the site's assemblage could have $p_{jt} = 0$ for a period t, depending on the site history's overlap with the ware's: $p_{jt} = 0$ if $[s_0 + (t-1)\,d] \ge w_{j1}$, or $[s_0 + (t\,d)] \le w_{j0}$. p_{jt} reflects the conditional distribution implied by limiting ware j's history to its overlap with the site's inhabitation (s_0–s_1). The earliest and latest overlaps between the ware and site are

$$v_{j0} = \max\{s_0, w_{j0}\}, \text{ and } v_{j1} = \min\{s_1, w_{j1}\}.$$

Then the relevant conditional distribution F^* is

$$F^*(x) = \left[F(x) - F(v_{j0})\right] / \left[F(v_{j1}) - F(v_{j0})\right], \text{ for } v_{j0} \le x \le v_{j1}.$$

In obtaining the apportioning probability p_{jt} for any period t, we allow the ware's start and/or end dates to not be multiples of d. (For convenience, the site dates are.) With

$$q_{jt0} = \max\{[s_0 + (t-1)\,d],\ w_{j0}\}, \text{ and}$$
$$q_{jt1} = \min\{[s_0 + (t\,d)],\ w_{j1}\},$$

the probability of apportioning a ware j sherd to period t is

$$\begin{aligned} p_{jt} &= F^*\left(q_{jt1}\right) - F^*\left(q_{jt0}\right) \\ &= \left[F\left(q_{jt1}\right) - F\left(q_{jt0}\right)\right] / \left[F(v_{j1}) - F(v_{j0})\right]. \end{aligned}$$

A. 2. Uniform distribution

In this case $F(x) = (x - w_{j0})/(w_{j1} - w_{j0})$, for values (years) $w_{j0} \le x \le w_{j1}$, so that

$$p_{jt} = \left(q_{jt1} - q_{jt0}\right) / \left(v_{j1} - v_{j0}\right).$$

A. 3. Truncated standard normal distribution

The general expressions can be modified for use with the truncated standard normal. Positive value z^* indicates the (symmetric) truncation points: $z^* = 2$ for truncation at values 2 and -2. The midpoint and length of the ware's lifespan are $m_j = (w_{j0} + w_{j1})/2$, and $g_j = (w_{j1} - w_{j0})$. Transforming values above leads to convenient expressions involving $\Phi(x)$, the standard normal's cumulative distribution function. We transform v_{j0} and v_{j1} as

$$z_{j0} = (v_{j0} - m_j) / \left(g_j/2z^*\right), \text{ and } z_{j1} = (v_{j1} - m_j) / \left(g_j/2z^*\right).$$

Note that $z_{j0} = -z^*$ if $v_{j0} = w_{j0}$, and $z_{j1} = z^*$ if $v_{j1} = w_{j1}$. q_{jt0} and q_{jt1} are similarly transformed into Z values:

$$z_{jt0} = \left(q_{jt0} - m_j\right) / \left(g_j/2z^*\right), \text{ and}$$
$$z_{jt1} = \left(q_{jt1} - m_j\right) / \left(g_j/2z^*\right).$$

Replacing $F(x)$ with $\Phi(x)$ in the expressions above,

$$p_{jt} = \left[\Phi(z_{jt1}) - \Phi(z_{jt0})\right] / \left[\Phi(z_{j1}) - \Phi(z_{j0})\right].$$

Table A.1 shows the necessary values and subsequent calculations for the example of Corrugated Brown Ware with the truncated standard normal distribution that is discussed in the main text.

Table A.1
Apportioning of 333 sherds of Corrugated Brown Ware ($j = 2$) at Bayless Ranch Ruin using truncated ($z^* = 2$) standard normal curve.

Values	Period		
	1200–1250 ($t = 1$)	1250–1300 ($t = 2$)	1300–1350 ($t = 3$)
q_{2t0}, q_{2t1}	1200, 1250	1250, 1300	1300, 1350
z_{2t0}, z_{2t1}	0.667, 1.000	1.000, 1.333	1.333, 1.667
$\Phi(z_{2t1}) - \Phi(z_{2t0})$	$\Phi(1.000) - \Phi(0.667) =$ $(0.8414 - 0.7475) = 0.0939$	$\Phi(1.333) - \Phi(1.000) =$ $(0.9088 - 0.8414) = 0.0674$	$\Phi(1.333) - \Phi(1.000) = (0.9522 - 0.9088) = 0.0434$
p_{2t}	0.4587	0.3293	0.2120
$N_2\ p_{2t}$	$333 \times 0.4587 = 152.747$	$333 \times 0.3293 = 109.657$	$333 \times 0.2120 = 70.596$

Also: $s_0 = 1200$, $s_1 = 1350$, $d = 50$, $T = 3$, $w_{20} = 800$, $w_{21} = 1400$; $v_{20} = 1200$, $v_{21} = 1350$, $m_2 = 1100$, $g_2 = 600$.
$z_{20} = (1200 - 1100)/(600/4) = 0.667$, $z_{21} = (1350 - 1100)/(600/4) = 1.667$.
$\Phi(z_{21}) - \Phi(z_{20}) = \Phi(1.667) - \Phi(0.667) = (0.9522 - 0.7475) = 0.2047$.
$z_{2t0} = (q_{2t0} - m_2)/(g_2/2z^*)$, $z_{2t1} = (q_{2t1} - m_2)/(g_2/2z^*)$, $p_{2t} = [\Phi(z_{2t1}) - \Phi(z_{2t0})]/[\Phi(z_{21}) - \Phi(z_{20})]$.

References

Brainerd, G.W., 1951. The place of chronological ordering in archaeological analysis. American Antiquity 16, 301–313.

Carlson, D.L., 1983. Computer analysis of dated ceramics: estimating dates and occupational ranges. Southeastern Archaeology 2, 8–20.

Clark, J.J., Lyons, P.D. (Eds.), 2012. Migrants and Mounds: Classic Period Archaeology of the Lower San Pedro Valley. Anthropological Papers No. 45. Center for Desert Archaeology, Tucson.

Cowgill, G., 1990. Why Pearson's *r* is not a good similarity coefficient for comparing collections. American Antiquity 55, 512–521.

Deming, W.E., Stephan, F.F., 1940. On a least squares adjustment of a sampled frequency table when the expected marginal totals are known. Annals of Mathematical Statistics 11, 427–444.

Goetze, C.E., Mills, B.J., 1993. Ceramic chronometry. In: Mills, B.J., Goetze, C.E., Zedeño, M.N. (Eds.), Across the Colorado Plateau: Anthropological Studies Along the Transwestern Pipeline Route. Ceramic Studies, Maxwell Museum of Anthropology and Office of Contract Archeology, vol. XVI. University of New Mexico, Albuquerque, pp. 87–150.

Hill, J.B., Clark, J.J., Doelle, W.H., Lyons, P.D., 2004. Prehistoric demography in the Southwest: migration, coalescence and Hohokam population decline. American Antiquity 69, 689–716.

Kohler, T.A., Blinman, E., 1987. Solving mixture problems in archaeology: analysis of ceramic materials for dating and demographic reconstruction. Journal of Anthropological Archaeology 6, 1–28.

Mills, B.J., 1989. Integrating analyses of vessels and sherds through models of ceramic assemblage formation. World Archaeology 21, 133–147.

Ortman, S.G., 2003. Artifacts. In: Kuckelman, K.A. (Ed.), The Archaeology of Yellow Jacket Pueblo (Site 5MT5): Excavations at a Large Community Center in Southwestern Colorado. http://www.crowcanyon.org/yellowjacket.

Ortman, S.G., Varien, M.D., Gripp, T.L., 2007. Empirical Bayesian methods for archaeological survey data: an application from the Mesa Verde region. American Antiquity 72, 241–272.

Pauketat, T.R., 1989. Monitoring Mississippian homestead occupation span and economy using ceramic refuse. American Antiquity 54, 288–310.

Ringrose, T.J., 1992. Bootstrapping and correspondence analysis in archaeology. Journal of Archaeological Science 19, 615–629.

Robinson, W.S., 1951. A method for chronologically ordering archaeological deposits. American Antiquity 16, 293–301.

Steponaitis, V.P., Kintigh, K.W., 1993. Estimating site occupation spans from dated artifact types: some new approaches. In: Stoltman, J. (Ed.), Archaeology of Eastern North America: Papers in Honor of Stephen Williams. Archaeological Report No. 25. Mississippi Department of Archives and History, Jackson, pp. 349–361.

Sullivan III, A.P., 2008. Ethnoarchaeological and archaeological perspectives on ceramic vessels and annual accumulation rates of sherds. American Antiquity 73, 121–135.

Varien, M.D., Mills, B.J., 1997. Accumulations research: problems and prospects for estimating site occupation span. Journal of Archaeological Theory and Method 4, 141–191.